Mathematical Lies and the Lying Liars who Teach Them

— or —

The Peano Postulates have been Drinking, Not Me

0:00 – 0:15

We Lie to Students

0:15 – 0:30

Precalculus: A Good Time to Stop Lying

0:30 – 0:45

Key Ideas

- Closure
- Identity elements
- Inverse elements

0:45 – 1:00

Back to the Beginning: Counting

1:00 – 1:15

Peano Postulates

1. $0 \in \mathbb{N}$
2. $n \in \mathbb{N} \Rightarrow n+1 \in \mathbb{N}$

1:15 – 1:30

Peano Postulates

1. $0 \in \mathbb{N}$
2. $n \in \mathbb{N} \Rightarrow n+1 \in \mathbb{N}$
3. $n+1=0 \Rightarrow n \notin \mathbb{N}$
4. $n+1=m+1 \Rightarrow n=m$

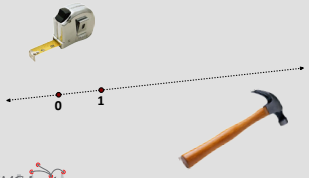
1:30 – 1:45

Peano Postulates

1. $0 \in \mathbb{N}$
2. $n \in \mathbb{N} \Rightarrow n+1 \in \mathbb{N}$
3. $n+1=0 \Rightarrow n \notin \mathbb{N}$
4. $n+1=m+1 \Rightarrow n=m$
5. $0 \in S$ and $\forall n \in S, n+1 \in S \Rightarrow S=\mathbb{N}$

1:45 – 2:00

Geometric Construction



2:00 – 2:15

Geometric Construction

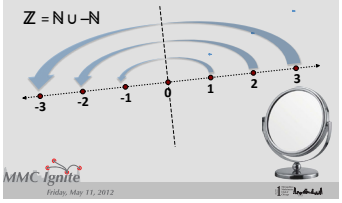
2:15 – 2:30

The Natural Numbers $\mathbb{N}$

2:30 – 2:45

Natural Numbers and their Opposites

$\mathbb{Z} = \mathbb{N} \cup -\mathbb{N}$



2:45 – 3:00

The Integers $\mathbb{Z}$

3:00 – 3:15

Ratios of Integers

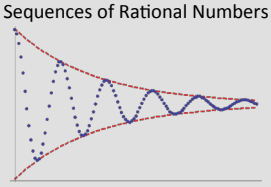
$\mathbb{Q} = \{ {}^{a}/_{b} \text{ with } a,b \in \mathbb{Z} \text{ and } b \neq 0 \}$

3:15 – 3:30

The Rational Numbers

3:30 – 3:45

Sequences of Rational Numbers



3:45 – 4:00

The Real Numbers

4:00 – 4:15

Square Roots of Negative Numbers

4:15 – 4:30

The Complex Numbers

4:30 – 4:45

We're Still Lying

4:45 – 5:00

Time's Up!

- Apologies to Al Franken and Tom Waits for the titles.
- **Isaac Greenspan**
  - isaac@isaacgreenspan.com
  - teacher, editor, writer, consultant
  - http://talks.isaacgreenspan.com/MMCIgnite2012.pdf

5:00

Mathematical Lies and the Lying Liars who Teach Them

— or —

The Peano Postulates have been Drinking, Not Me

MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago

1913

•

We Lie to Students

MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- can't subtract bigger number from smaller
- can't divide 10 by 3
- can't take the square root of negative numbers

Precalculus: A Good Time to Stop Lying

MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- ready to talk about complex numbers
- "algebraically closed" is meaningful and somewhat provable
- at a place where, essentially, we get to a set (complexes) that is closed under all the machinery the students have and will have for some time to come

Key Ideas

- Closure
- Identity elements
- Inverse elements

•

Back to the Beginning: Counting

MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- go back to before we started lying
- define things with some degree of care

Peano Postulates

1. $0 \in \mathbb{N}$
2. $n \in \mathbb{N} \Rightarrow n + 1 \in \mathbb{N}$

MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- 0 is a natural number (could start with 1, but 0 is nicer in a few ways)
- if n is a natural number, then the "next" number is also a natural number—"next" means adding 1

Peano Postulates

1. $0 \in \mathbb{N}$
2. $n \in \mathbb{N} \Rightarrow n + 1 \in \mathbb{N}$
3. $n + 1 = 0 \Rightarrow n \notin \mathbb{N}$
4. $n + 1 = m + 1 \Rightarrow n = m$

- 0 doesn't come after any natural number
- if the "next" number from n and the "next" number from m are the same, then n and m are the same

Peano Postulates

1. $0 \in \mathbb{N}$
2. $n \in \mathbb{N} \Rightarrow n + 1 \in \mathbb{N}$
3. $n + 1 = 0 \Rightarrow n \notin \mathbb{N}$
4. $n + 1 = m + 1 \Rightarrow n = m$
5. $0 \in S$ and $\forall n \in S, n + 1 \in S$
   $\Rightarrow S = \mathbb{N}$

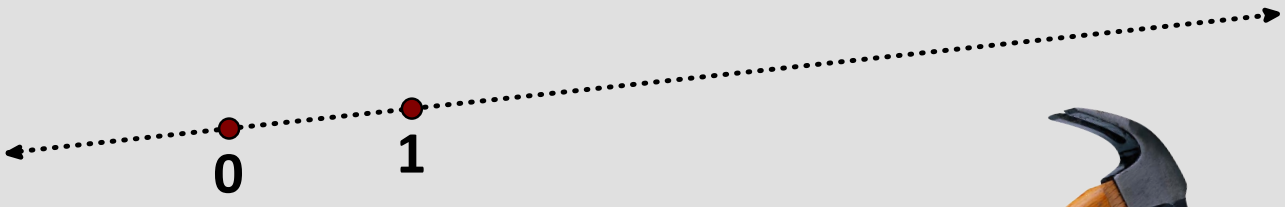
MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago

- induction!

Geometric Construction



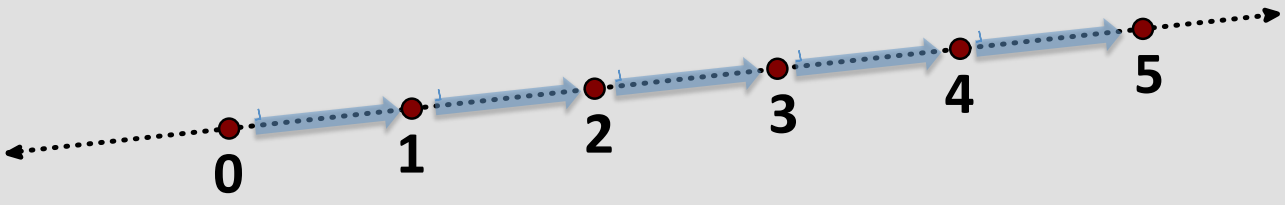
MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- given a point 0 and a vector that takes 0 to 1 (alternately, the point 1)

Geometric Construction



MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- "n + 1" is translating n by the 1 vector (defined by the translation mapping 0 to 1)

The Natural Numbers $\mathbb{N}$

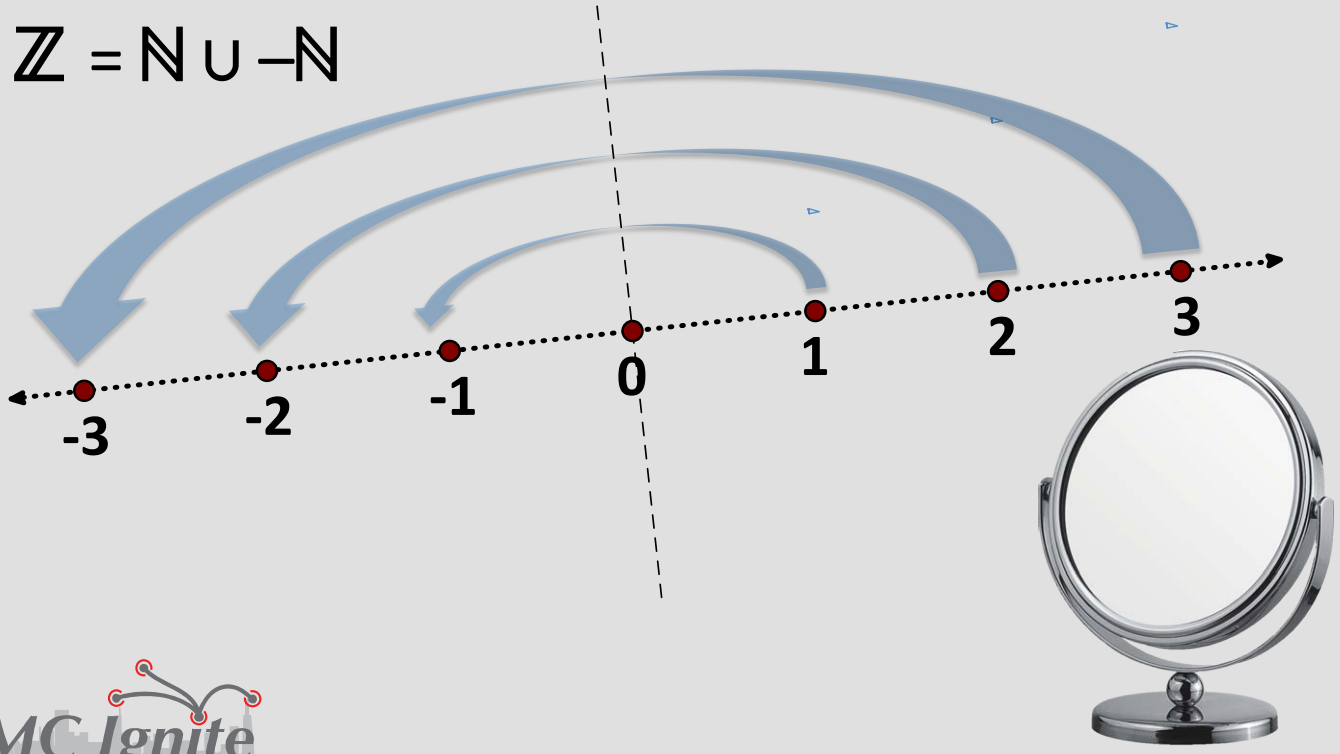
MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- closed under addition (and multiplication), but not subtraction (nor division)

Natural Numbers and their Opposites



MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago

- opposites = additive inverses
- algebraic definition of subtraction
- reflection through 0

The Integers $\mathbb{Z}$

MMC Ignite

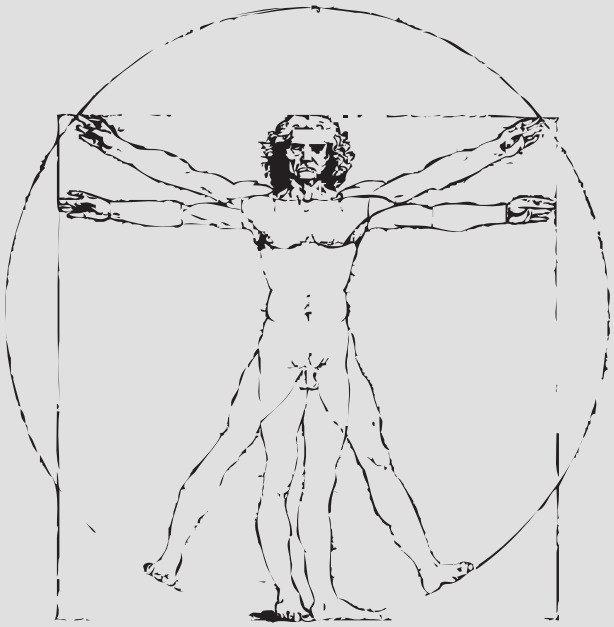
Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- closed under addition and subtraction

Ratios of Integers

$\mathbb{Q} = \{ {}^{a}/_{b}$ with $a,b \in \mathbb{Z}$ and $b \neq 0\}$



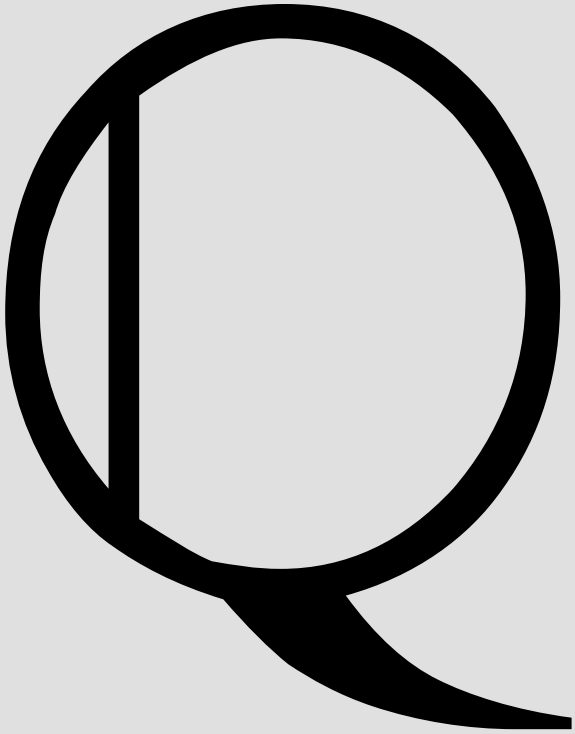
MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- integers are closed under multiplication, but not division
- consider ratios of integers
- geometrically, scaling/dilation (which is constructible)

The Rational Numbers



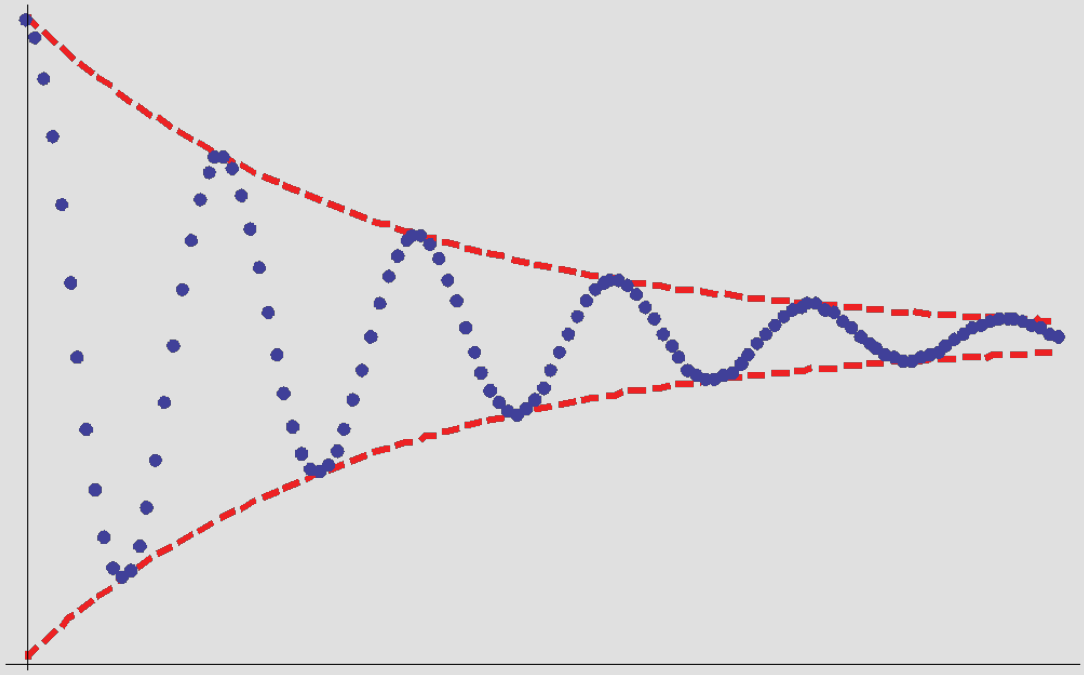
MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- gained multiplicative inverses, closure under division (except by 0)
- lost next/previous
- dense

Sequences of Rational Numbers



MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- "Cauchy" sequences give us a technical way of saying "sequences that converge at all"
- consider limits of all Cauchy sequences of rational numbers
- not exactly constructible... but this goes from discrete points to the continuous line

The Real Numbers

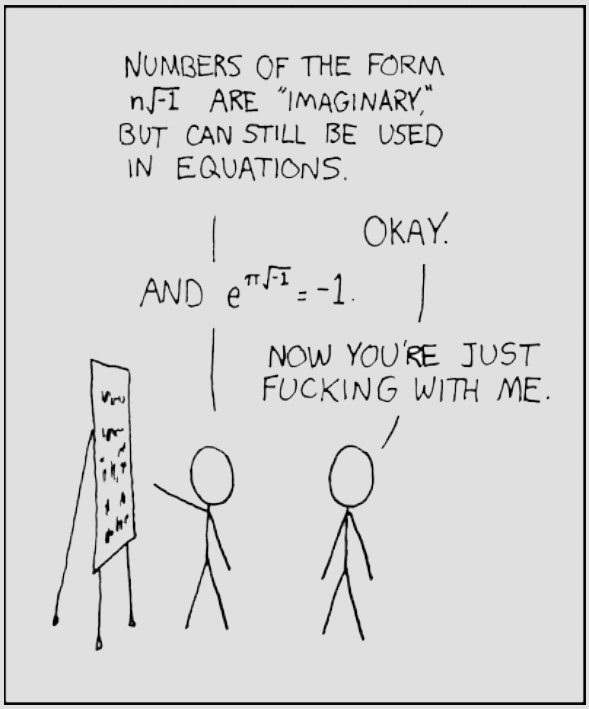
MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago

- closed under limits of Cauchy sequences
- no longer countably infinite
- lost nice representations
- alternately: Dedekind cuts

Square Roots of Negative Numbers



Comic by Xkcd, http://xkcd.com/179/

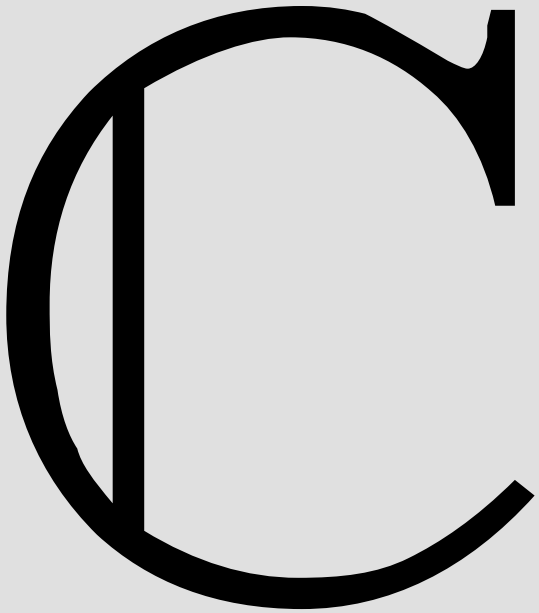
MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- $i=\sqrt{-1}$
- a+bi where a, b are real
- take our number line, rotate it 90°, and make a coordinate plane

The Complex Numbers



MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- algebraically closed (Fundamental Theorem of Algebra)
- lost inequality-type ordering/trichotomy

We're Still Lying

MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- more by omission than outright lying now
- compactification point at infinity (Riemann sphere)
- quaternions

Time's Up!

- Apologies to Al Franken and Tom Waits for the titles.
- **Isaac Greenspan**
  - **isaac@isaacgreenspan.com**
  - **teacher, editor, writer, consultant**
  - **http://talks.isaacgreenspan.com/MMCIgnite2012.pdf**

MMC Ignite

Friday, May 11, 2012

Metropolitan Mathematics Club of Chicago 1913

- thank you all
- slides, along with my notes, are at the URL shown
- MMCIgnite2012-slides-only.pptx is the PowerPoint; MMCIgnite2012-slides-only.pdf is a PDF of that PowerPoint