

Trigonometric Identities: Techniques, Examples, Connections

Isaac Greenspan

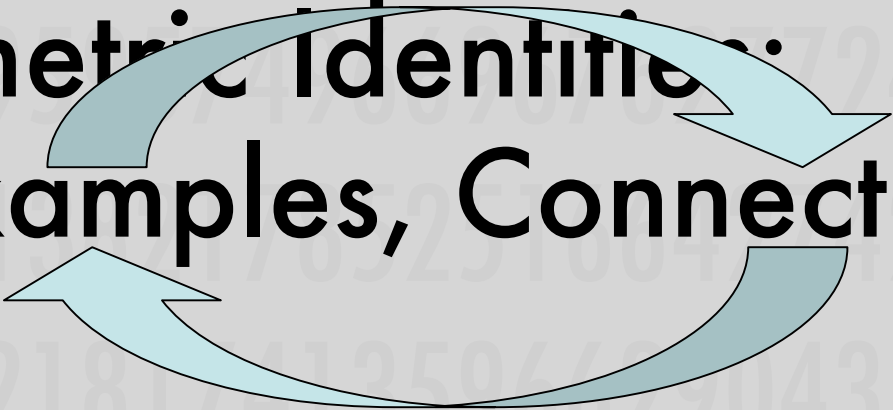
Editor, UCSMP

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MMC Conference of Workshops
Lemont High School, Lemont, IL
Saturday, January 26, 2008

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In case you get bored:

Prove: if $a + b + c = \pi$ and n is an integer, then
 $\tan(na) + \tan(nb) + \tan(nc) = \tan(na) \tan(nb) \tan(nc)$

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If that doesn't last you through the whole 75 minutes, find and prove a similar identity involving cotangent.

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Need more? How about the other 4 trig functions?

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Techniques

Prove $1 + \tan^2 x = \sec^2 x$ by:

- Rewriting one side using definitions, known identities, and algebraic properties until it equals the other side.
- Rewriting each side independently until equal expressions are obtained.
- Beginning with a known identity and transform it until the desired identity appears.
- Transforming both sides of the equation to be proved using *reversible steps* until a known identity appears.

[UCSMP Precalculus and Discrete Mathematics, 2nd ed.]

Techniques

Proof Technique Summary

- a. LHS = ... = RHS
- b. LHS = ... = \square ; RHS = ... = \square
- c. Known Identity \Rightarrow Target Identity
- d. Target Identity \Rightarrow Known Identity
(with reversible steps)

Techniques

For each of the following identities, choose a method of proof.

$$\frac{1-\sin t}{\cos t} = \frac{\cos t}{1+\sin t}$$

$$\frac{1}{1+\sin m} + \frac{1}{1-\sin m} = 2\sec^2 m$$

$$\frac{\sin t + \cos t}{\sec t + \csc t} = \frac{\cos t}{\csc t}$$

$$\frac{1}{\sqrt{1+\sin x}} = |\sec x| \sqrt{1-\sin x}$$

$$\tan t + \cot t = \sec t \csc t$$

$$\sin^4 t - \cos^4 t = \sin^2 t - \cos^2 t$$

$$1 - 2\cos^2 r = 2\sin^2 r - 1$$

$$\tan^4 p - 1 = \sec^4 p - 2\sec^2 p$$

[Dolciani Modern Introductory Analysis]

Connections

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Fortunately, multiplying a complex number $x + iy$ by $\cos \theta + i \sin \theta$ rotates (x, y) by θ about the origin:

$$(x + iy)(\cos \theta + i \sin \theta) = x \cos \theta - y \sin \theta + i(x \sin \theta + y \cos \theta)$$

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$$\text{and } R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$

Connections

Rotating by α then by β should be the same as rotating by $(\alpha + \beta)$. Use this fact to derive identities for sine and cosine of $(\alpha + \beta)$.

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Did you use matrices or did you use complex numbers?

Connections

Hyperbolic Trigonometry:

- Refer to your handout
- Look at review of circular trigonometry—you need this particular perspective on circular trigonometry for the derivation of hyperbolic trigonometry.
- Go through derivation of hyperbolic trigonometry. You may want to let a CAS do the heavy lifting for you and/or skip over some of the calculus involved.

Connections

Hyperbolic Trigonometry—Executive Summary

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

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Use the fact that $e^{ix} = \cos x + i \sin x$ to find formulae for $\cos x$ and $\sin x$ in terms of exponentials.

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Use your formulae to prove $\cos 2x = 2\cos^2 x - 1$
(hint: start with the right side).

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Use your formulae to evaluate $\int \cos^2 x \, dx$ without using the normal reduction formula.

Write $\cosh ix$ and $\sinh ix$ in terms of $\cos x$ and $\sin x$.

Examples

$\sin(nx)$ and $\cos(nx)$

Examples

If $\alpha + \beta + \gamma = \pi \dots$

2.7182818284590452353602874713526624
97757247093699959574966967627724076
63035354759457138217852516642742746
63919320030599218174135966290435729
00334295260595630738132328627943490
76323382988075319525101901157383418
79307021540891499348841675092447614
60668082264800168477411853742345442
43710753907774499206955170276183860

Yes, the background is the first several digits of e .

2.7182818284590452353602874713526624
97757247093699959574966967627724076
63035354759457138217852516642742746
63919320030599218174135966290435729
00334295260595630738132328627943490
76323382988075319525101901157383418
79307021540891499348841675092447614
60668082264800168477411853742345442
43710753907774499206955170276183860

Yes, the background is the first several digits of e .

But, really, you should be wondering why this presentation isn't over yet...

