Unexpectedly Determined

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Bouncing Ping-Pong Ball

Given position data for a projectile, time data can be (roughly) derived because the acceleration due to gravity is known and this determines one coefficient in the equation for vertical position in terms of time. It is possible to determine parametric equations for the position of, for instance, a bouncing ball from a stroboscopic photo of the bouncing ball without knowing the time interval between flashes.

- Open Geometer's Sketchpad.
- Go to the preferences and change the precision for scalars to thousandths.
- Open a web browser and go to http://talks.isaacgreenspan.com/ and find the link for "UnexpectedlyDetermined_Links.html."
- Right-click on one of the bouncing-ball photos and select "Copy Image."
- Go to Sketchpad, right-click, and select "Paste Picture."
- You may want to resize or reposition the picture. If you resize it, make sure to hold down the shift key while you resize to preserve the aspect ratio of the picture.
- Right-click on the picture, select "Properties...," uncheck "Arrow Selectable," and click "OK."
- Use the circle tool to create a circle, position it atop one image of the ball, and size it to the image of the ball.
- Select the circle only, go to the "Measure" menu, and select "Radius."
- The actual radius of a ping-pong ball is 20mm, or $\frac{1}{50}$ of a meter. That is, 1 meter is 50 times the measurement just taken. Go to the "Measure" menu, select "Calculate...," select the radius measurement, enter "*50" to multiply it by 50, and click "OK."
- With the newly-calculated 1-meter measurement selected, go to the "Graph" menu and select "Define Unit Distance" to define a coordinate system using this measurement as the unit distance (that is, create a coordinate system where the units are meters in the photograph).
- Move the origin to a convenient location.
- On your existing circle, hide the point on the circle (right-click, "Hide Point").
- Select the center of the circle, go to the "Measure" menu, and select "Abscissa (x)."
- Select the center of the circle, go to the "Measure" menu, and select "Ordinate (y)."
- Select the abscissa and ordinate measurements, then go to the "Graph" menu and select "Tabulate."
- Move the circle (grab the circle itself, not any point) to the first image of the ball in the arc you want to model.
- Double-click on the table heading to add a row to the table for the location of the first image.
- Move the circle to each subsequent image of the ball on the arc you want to model, double-clicking on the table heading to add the data to the table.

- Select the table and copy it.
- Open the TI-Nspire software.
- Click to add an application and select "Add Lists & Spreadsheet."
- Paste the data into the spreadsheet. You may need to delete a row of headings and a duplicate row at the bottom.
- Name the columns "xlist" and "ylist" (in the topmost row above the spreadsheet data).
- Name the third column "time" and put your best guess of the time interval (in seconds) into the first row.
- In the next cell down, enter the formula "=c1+$c1", then right-click on the cell, select "Fill Down," use the down arrow key to select the rest of the column, and press enter.
- Insert a "Data & Statistics" page.
- Click on "Click to add variable" at the bottom edge of the page and select "time."
- Click on the left edge of the page where "Click to add variable" had been and select "ylist."
- Click on "Analyze" (4th button from the left in the lower toolbar), select "Regression," and select "Show Quadratic."
- The resulting quadratic equation expresses the height of the ball (in meters) as a function of time (in seconds) based on the time interval you guessed. The coefficient of the quadratic term should be half the acceleration due to gravity ($-9.8\frac{\text{m}}{\text{s}^2}$), so –4.9. If your coefficient is not –4.9, go back to the spreadsheet page and change your guess for the time interval. When you return to the statistics page, the plot, regression curve, and equation will all be updated to your new time interval. Repeat as necessary.
- To determine an equation for horizontal position in terms of time, insert an additional statistics page, put "time" on the horizontal axis and "xlist" on the vertical axis, and apply a linear regression.